

1104-01

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Degree (Part-I) Examination, 2020

(Honours)

MATHEMATICS

[Paper : First]

[PPU-D-I(H)-MATH-1]

Time : Three Hours]

[Maximum Marks : 100

Note: Candidates are required to give their answers in their own words as far as practicable. The figure in the margin indicate full marks. The questions are of equal value.

1. (i) If A, B, C are non-empty set, then :

(a) $A \cap (B - C) = (A \cap B) - (A \cap C)$

(b) $A \cap (B - C) = (A \cap B) - C$

(c) $A \cup (B - C) = (A \cup B) \cap (A \cup C)$

(d) $A \cup (B - C) = (A \cup B) - (A \cup C)$

(ii) Let R be a relation on N defined by :

$R = \{(a, b) : a, b \in N \text{ and } a = b^2\}$, then :

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(1)

[P.T.O.]

(a) R is reflexive, symmetric but not transitive

(b) R is symmetric but not reflexive and transitive

(c) R is not reflexive, symmetric and transitive

(d) R is reflexive, symmetric and transitive

(iii) If A and B are square symmetric matrices, then (A+B) is :

(a) a symmetric matrix

(b) a skew symmetric matrix

(c) Neither a symmetric matrix nor a skew symmetric

(d) None of the above

(iv) If $A = \begin{bmatrix} \cos \theta & \sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$, then AA^T is :

(a) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

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(2)

(b) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(v) The equation $x^4 - x^3 + x^2 - 1 = 0$ has :

- (a) One real root and three imaginary roots
- (b) Two real roots and two imaginary roots
- (c) All roots are imaginary
- (d) All roots are real

(vi) If A be a non-singular matrix of order n, then $\det \text{adj } A$ is :

- (a) $(\det A)^n$
- (b) $\det A$
- (c) $(\det A)^{n-1}$
- (d) $(\det A)^{n-2}$

(vii) If the feasible region of a LPP is empty, then solution is :

- (a) infeasible
- (b) Unbounded
- (c) Alternative
- (d) None of the above

(viii) In simplex method we addvariables in case of "="

- (a) Slack variable
- (b) Surplus variable
- (c) Artificial variable
- (d) None of the above

(ix) Set of natural numbers is :

- (a) Uncountable
- (b) Countable
- (c) Infinitely countable
- (d) Finitely countable

(x) If A is a real skew symmetric matrix and $A^2 + I = 0$, then A is :

- (a) Symmetric
- (b) Unitary
- (c) Orthogonal
- (d) None of the above

Group-A

2. (a) Define union and intersection of two sets A & B and show that $(A \cup B)' = A' \cap B'$.

(b) Let R be the relation on set Z of all integers defined by $(x, y) \in R \Rightarrow x - y$ is divisible by n, then prove that R is an equivalence relation.

3. (a) If A & B are two countable sets, then prove that $A \cup B$ is countable.

(b) Find all the roots of $x^7 = 1$.

4. (a) If $x = \log_e \tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$, Prove that :

(i) $\tanh\left(\frac{x}{2}\right) = \tan\left(\frac{\theta}{2}\right)$

(ii) $\cos\theta \cdot \cosh x = 1$

(b) Prove that $\frac{2.4.6.8 \dots (2n)}{1.3.5.7 \dots (2n-1)} = \sqrt{n\pi}$ where n is very large

Group-B

5. (a) For the system of equations :

$$x + y + z = 6$$

$$x + 2y + 3z = 10$$

$$x + 2y + \lambda z = \mu$$

Find the value of λ, μ so that the system has :

- (i) Unique solution
- (ii) Infinite solution
- (iii) No solution

(b) If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, compute A^{-2} .

6. (a) Find the inverse of the matrix :

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$$

(b) Solve the given system of linear equations :

$$2x + 5y + 2z = -38$$

$$3x - 2y + 4z = 17$$

$$-6x + y - 7z = -12$$

Group-B

7. (a) Solve the following LPP graphically :

$$\text{Max } Z = 7x_1 + 5x_2$$

$$\text{Subject to constraints } 2x_1 + 3x_2 \leq 20$$

$$3x_1 + x_2 \geq 10 \text{ and}$$

$$x_1, x_2 \geq 0$$

(b) Solve the following LPP graphically :

$$\text{Max } Z = 3x_1 - x_2$$

$$\text{Subject to constraints } 2x_1 + x_2 \leq 2$$

$$x_1 + 3x_2 \geq 3$$

$$x_2 \leq 4 \text{ and}$$

$$x_1, x_2 \geq 0$$

8. Solve the given LPP by simplex method :

$$\text{Max } Z = 2x_1 + 5x_2 + 7x_3$$

Subject to constraints

$$3x_1 + 2x_2 + 4x_3 \leq 100$$

$$x_1 + 4x_2 + 2x_3 \leq 100$$

$$x_1 + x_2 + 3x_3 \leq 100$$

$$\text{and } x_1, x_2, x_3 \geq 0.$$

Group-D

9. (a) Solve $x^4 - 4x^3 + 6x^2 + x - 10 = 0$

(b) Solve the given equation using Carden's method :

$$x^3 - 6x - 9 = 0$$

10. (a) State and prove the fundamental theorem of Algebra.

(b) If α, β, γ be the roots of equation

$$x^3 + qx + r = 0, \text{ find the equation whose roots are } \beta\gamma - \alpha^2, \gamma\alpha - \beta^2, \alpha\beta - \gamma^2.$$

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