1204-01 **Printed Pages : 12**

Degree (Part-I) Examination, 2020

(Subsidiary)

MATHEMATICS

[PPU-D-I(SUB)-MATH]

Time : Three Hours] [Maximum Marks : 100

Note : Answer **any six** questions including question **no 1,** which is **compulsory**, selecting at least one from each group.

1. *Answer all the following objective type questions :*

(i) If $f : R \longrightarrow R^+$ is the mapping defined by $f(x) = x^2$, then which one is true :

(a) f is injective

(b) f is surjective

(c) f is bijective

(d) None of these

(ii) Which of the following property of matrix multiplication is correct :

(a) Multiplication is not commutative in general

(b) Multiplication is associative

(c) Multiplication is distributive over addition

(d) All of the above

(iii) If $z = ax^3 + by^3$, where a and b are arbitrary constants, then $x\left(\frac{\partial z}{\partial x}\right)+y\left(\frac{\partial z}{\partial y}\right)$ is equal to :

(a) 0

(b) 3z

(c) –3z

(d) None of these

(iv) In a LPP, the solution of a variable can be made infinitely large without violating the constraints, the solution is :

(a) Infeasible

(b) Alternative

(c) Unbounded

(d) Bounded

(v) The reflection of the points (3, 1, 2) in the plane $x + 2y + z = 1$ is :

(a) (1, 3, 0)

(b) (1, 0, 3)

(c) (–3, 0, 1)

(d) (1, –3, 0)

(vi) The distance between the parallel planes $2x - 2y + z = 10$ and $4x + 4y + 2z = 2$ is :

(a) 9 units

(b) 3 units

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(c) $\sqrt{3}$ units

(d) 1 unit

(vii) The direction cosines of the line joining the points (2, 3, 1) and (3, –1, 2) are :

(a) $\frac{-1}{\sqrt{18}}, \frac{-4}{\sqrt{18}}, \frac{-1}{\sqrt{18}}$

(b) $\frac{-1}{\sqrt{18}}, \frac{-4}{\sqrt{18}}, \frac{1}{\sqrt{18}}$

(c) $\frac{1}{\sqrt{18}}, \frac{4}{\sqrt{18}}, \frac{1}{\sqrt{18}}$

(d) $\frac{1}{\sqrt{18}}, \frac{-4}{\sqrt{18}}, \frac{1}{\sqrt{18}}$

(viii) Which of the following series is divergent :

(a) $\sum_{n=1}^{\infty} \frac{1}{n^2}$

(b) $\sum_{n=1}^{\infty} \frac{1}{(\log n)^n}$

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(c) $\sum_{n=1}^{\infty} \log\left(\frac{n+1}{n}\right)$

(d) $\sum_{n=1}^{\infty} \frac{1}{n!}$

(ix) In the set $S = R - \{1\}$ under the binary operation 0 given by a 0 b = a + b + ab, the identity element is :

(a) 1

(b) 0

(c) –1

(d) does not exist

(x) i^i equal, (where $i^2 = -1$) :

(a) $e^{(4n+1)\frac{\pi}{2}}$

(b) $e^{(2n+1)\pi}$

(c) $e^{-(2n+1)\pi}$

(d) $e^{-(4n+1)\frac{\pi}{2}}$

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GROUP-A

2. (a) For any two sets A and B, prove that :

(i) $A \cup (A \cap B) = A$

(ii) $A \cap (A \cup B) = A$

(b) If R be a relation from X and Y, $R \subseteq X * Y$ and let A & B two subsets of X, hence show that :

$$R[A \cap B] \subseteq R[A] \cap R[B]$$

3. (a) Define group. Show that under the binary operation 0 defined on Q^* by a 0 $b = \frac{ab}{2}$ is a group.

(b) Prove that the subgroup of a cyclic group is also cyclic.

GROUP-B

4. (a) If A and B are symmetric matrices of same order and commute also, then show that $A^{-1}B$, AB^{-1} and $A^{-1}B^{-1}$ are also symmetric.

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(b) Find the inverse of the matrix :

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix}$$

5. (a) Solve the LPP graphically :

Max $Z = -x_1 + 2x_2$

Subject to constraints :

$x_1 - x_2 \leq -1,$

$0.5\, x_1 + x_2 \leq 2$

$x_1, x_2 \geq 0$

(b) Define rank of a matrix and find the rank of Matrix:

$$\begin{bmatrix} 1 & -1 & 0 & 2 \\ 2 & 0 & 1 & 3 \\ 2 & 4 & 3 & 1 \\ 3 & 5 & 4 & 2 \end{bmatrix}$$

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GROUP-C

6. (a) Prove by the use of De Moivre's theorem that the roots of the equation $(x-1)^n = x^n$, n being a positive integer are $\frac{1}{2}\left\{1 + i\cot\frac{r\pi}{n}\right\}$ where r = 1, 2, ..., n–1.

(b) Show that :

$$\sin\alpha = \alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + \ldots$$

7. (a) Define continuity and differentiability of a function at a point verify the continuity and differentiability of the function :

$$f(x) = \begin{cases} (1-x) & x < 1 \\ (1-x)(2-x) & 1 \leq x \leq 2 \\ (3-x) & x > 2 \end{cases}$$

(b) Define convergence of series. State and prove Cauchy's general principle of convergence of series.

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GROUP-D

8. (a) Find the condition that the line $y = mx + c$ be a tangent to the circle $x^2 + y^2 = a^2$.

(b) Prove that in an ellipse, the perpendicular from a focus upon any tangent and the line joining the centre of the ellipse of the point of contract meet on the correspondix direction.

9. (a) Show that the mid point of the line segment joining the focus to a moving point on the parabola $y^2 = 4ax$ moves on another parabola with directrix $x = 0$.

(b) Find the equation of tangent and normal to the $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (hyperbola) at the points ($a \sec\theta$, $b \tan\theta$).

GROUP-E

10. (a) Find the equation of the plane through the point (1, -2, 1) and perpendicular to two planes :

$2x - 2y + z = 0$ and $x - y + 2z = 4$

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(b) Find the equation of plane containing the lines :

$$\frac{5-x}{-4} = \frac{y-7}{4} = \frac{z+3}{-5} \text{ and } \frac{x-8}{7} = \frac{2y-8}{2} = \frac{z-5}{3}$$

11. (a) Find the angle between any two diagonals of a cube.

(b) Find the condition under which a general equation of second degree in two variable represent a circle.

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