1204-01 Printed Pages : 12

B.Sc. Part-I (Hons.) Examination, 2019

MATHEMATICS

[SUBSIDIARY]

[Paper : First]

[PPU-D-I (SUB)-MAT]

Time : Three Hours] [Maximum Marks : 100

Note : The figure in the margin indicate full marks. Answer any six questions, selecting one from each group. Question no. 1 is **compulsory**.

1. Multiple choice question: [2×10=20]

(a) If $f : Q \to Q$ is a mapping given by $f(x) = 2x + 3$, then which of the following is true :

(i) f is injective but not surjective

(ii) f is surjective but not injective

(iii) f is bijective

(iv) f is neither injective nor surjective

(b) The real part of $\sin(\log i^i)$ is :

(i) 1

(ii) -1

(iii) 0

(iv) $\frac{1}{2}$

(c) If A is any square matrix, then the value of A.adj A is :

(i) $\det(A).I$

(ii) $\det(A).A$

(iii) $\det(A).A^{-1}$

(iv) $\frac{1}{\det(A)}I$

(d) The image of the point (2, 1, 3) in the plane x + 2y - z +2 = 0 is :

(i) (1, 1, 4)

(ii) (-1, 1, 4)

(iii) (1, -1, 4)

(iv) (1, 1, -4)

(e) The direction cosines of a line which makes equal angle with coordinate axes are :

(i) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

(ii) $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

(iii) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}$

(iv) $\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

(f) The equation of the plane through the point (1, 0, -1) and perpendicular to the line segment joining points (3, 2, 5) and (4, 5, 2) is :

(i) x - 3y - 3z + 4 = 0

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(ii) x + 3y + 3z - 4 = 0

(iii) x + 3y - 3z - 4 = 0

(iv) x + 3y - 3z + 4 = 0

(g) Which of the following series is not convergent ?

(i) $\sum_{n=1}^{\infty} \frac{1}{n!}$

(ii) $\sum_{n=1}^{\infty} \frac{1}{(\log n)^2}$

(iii) $\sum_{n=1}^{\infty} \frac{4^n}{n!}$

(iv) $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n^2}\right)$

(h) In the group of non zero rational numbers under the binary operation 0 given by $a \circ b = \frac{ab}{3}$, the inverse of 9 is :

(i) 1

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(ii) $\frac{1}{9}$

(iii) 3

(iv) $\frac{1}{3}$

(i) If $y = \tan^{-1} x$, then $(1+x^2)\frac{d^2y}{dx^2}$ is equal to :

(i) $\frac{dy}{dx}$

(ii) $2x\frac{dy}{dx}$

(iii) $x\frac{dy}{dx}$

(iv) $-2x\frac{dy}{dx}$

(j) The set of all feasible solutions of a L.P.P. is always a :

(i) Convex set

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(ii) Open set

(iii) Closed set

(iv) Unbounded set

GROUP-A **[16]**

2. (a) For any two sets A and B, prove that $A = (A \cap B) \cup (A - B)$.

   (b) If $f : X \to Y$ be a mapping and A, B be two subsets of X, then show that $f(A \cap B) \subseteq f(A) \cap f(B)$.

3. (a) Define a group. Show that for a given positive integer n, nth root of unity form a multiplicative group.

   (b) Define integral domain and give an example of an integral domain which is not a field.

GROUP-B **[16]**

4. (a) If A and B are square matrices of same order and A is symmetric, then show that $B^T AB$ is also symmetric.

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(b) Find the inverse of the matrix $\begin{bmatrix} 2 & 5 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$.

5. (a) Define rank of a matrix and find the rank of the matrix $\begin{pmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 2 & 2 \\ 1 & 1 & -2 & 0 \end{pmatrix}$.

(b) Solve the L.P.P. graphically :

Max $z = 6x_1 + 5x_2$

subject to $x_1 + x_2 \le 10$

$2x_1 + x_2 \le 15$

$x_2 \le 8$

$x_1, x_2 \ge 0$

GROUP-C [16]

6. (a) Show that any differentiable function is continuous. Give an example of a function

continuous on $\mathbb{R}$ but not differentiable exactly at two points.

(b) Define convergence of a sequence. Show that the sum of two convergence sequences is also a convergence sequence.

7. (a) Using Demoivre's theorem prove that :

$$\sin 6\theta = 6\cos^5\theta\sin\theta - 20\cos^3\theta\sin^3\theta + 6\cos\theta\sin^3\theta$$

(b) Show that for $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$,

$$\tan\alpha = \alpha + \frac{\alpha^3}{3} + \frac{2}{15}\alpha^5 + \ldots\ldots$$

GROUP-D [16]

8. (a) Find the condition that the line y = mx + c be a tangent line to the parabola $y^2 = 4ax$.

(b) Show that the point of intersection of two mutually perpendicular tangent lines to an ellipse always lies on a circle.

9. (a) Show that there are at most four normals be drawn to a hyperbola from any point.

(b) Find the condition under which a general equation of second degree in two variable represent a circle.

GROUP-E [16]

10. (a) Find the angle between two diagonals of a cube.

(b) If α, β, γ, δ be angles made by a given line with four diagonals of a cube, then show that

$$\sin^2\alpha+\sin^2\beta+\sin^2\gamma+\sin^2\delta=\frac{8}{3}.$$

11. (a) Find the equation of the plane through the point (-1, 3, 1) and perpendicular to the line 2x + 3y + 4z = 5, 3x + 4y + 5z = 6.

(b) Find the equation of the plane containing the lines

$$\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1} \text{ and } \frac{x}{1}=\frac{y-7}{-3}=\frac{z+7}{2}.$$

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